

Suspensions in wild topology

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In general, the calculation of higher homotopy groups of suspensions and reduced suspensions is an amazingly difficult endeavour, and little is known, even for simple spaces like spheres. For the fundamental group, on the other hand, the situation is more clearly laid out as long as one only considers CW-complexes: $\pi_1 \Sigma X$ is just a free group with the number of connected components of X minus one as its rank.

This however belies the rich structure and great variety of groups that can be attained if arbitrary spaces are considered. For example, the *Hawaiian earring* with its rather complicated fundamental group is the reduced suspension of $\omega + 1$, i.e. a countable set with one limit point. One might be inclined to assume that $\pi_1 \Sigma X$ is at least completely determined by the path components $\pi_0 X$, as is true for the ordinary suspension, perhaps additionally equipped with an appropriate topology – but even that cannot capture the entire variability. As a second example, the reduced suspension of the topologist's sine curve (which has two path components) is known as the *harmonic archipelago* and its fundamental group contains the rationals \mathbb{Q} as a subgroup.

We will relate algebraic properties of $\pi_1 \Sigma X$, such as *not* containing divisible elements, to being isomorphic to some $\pi_1 \Sigma Y$ with Y a totally path disconnected space, and to a purely topological condition on X .

(Joint work with Samuel Corson.)