LIST OF INTEGERS OF SMALL NORM IN SIMPLEST CUBIC FIELDS OVER $\mathbb{Q}(\sqrt{-D})$

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Let $k := \mathbb{Q}(\sqrt{-D})$ be an imaginary quadratic number field and $\mathbb{Z}_k$ be the corresponding ring of integers. For $t \in \mathbb{Z}_k$ let be

$$f_t(x) := x^3 - (t-1)x^2 - (t+2)x - 1$$

and $\alpha = \alpha^{(1)}$ be a root of f_t. Let be $\gamma \in \mathbb{Z}_k[\alpha]$. For all t with $\Re t \leq -\frac{1}{2}$ and $\Im t > 0$

$$|N_{k(\alpha)/k}(\gamma)| \leq |2t+1|$$

implies that γ is associated to an integer in $\mathbb{Z}_k$ or

$$\gamma = \mu\beta\alpha^{b_1}(\alpha+1)^{b_2} \tag{1}$$

where μ is a unit of $\mathbb{Z}_k$, $b_1, b_2 \in \mathbb{Z}$ and β is either an element of the triple $\{\alpha-1, -(2\alpha+1), \alpha+2\}$ (satisfying $N_{k(\alpha)/k}(\beta) = 2t+1$) or an element of the triples presented in the list $\mathcal{L}(t,\alpha,\beta)$ below.

(For details we refer the reader to [1].)

Let

$M_3 := \{-4+9i\sqrt{2}, -3+9i\sqrt{2}, -2+9i\sqrt{2}, -1+9i\sqrt{2}, -5+8i\sqrt{2}, -4+8i\sqrt{2}, -3+8i\sqrt{2}, -2+8i\sqrt{2}, -1+8i\sqrt{2}, -6+7i\sqrt{2}, -5+7i\sqrt{2}, -4+7i\sqrt{2}, -3+7i\sqrt{2}, -2+7i\sqrt{2}, -1+7i\sqrt{2}, -6+6i\sqrt{2}, -5+6i\sqrt{2}, -4+6i\sqrt{2}, -3+6i\sqrt{2}, -2+6i\sqrt{2}, -1+6i\sqrt{2}, -6+5i\sqrt{2}, -5+5i\sqrt{2}, -4+5i\sqrt{2}, -3+5i\sqrt{2}, -2+5i\sqrt{2}, -1+5i\sqrt{2}, -6+4i\sqrt{2}, -5+4i\sqrt{2}, -4+4i\sqrt{2}, -3+4i\sqrt{2}, -2+4i\sqrt{2}, -1+4i\sqrt{2}, -5+3i\sqrt{2}, -4+3i\sqrt{2}, -3+3i\sqrt{2}, -2+3i\sqrt{2}, -1+3i\sqrt{2}, -4+2i\sqrt{2}, -3+2i\sqrt{2}, -2+2i\sqrt{2}, -1+2i\sqrt{2}\}$

and

$M_4 := \{-\frac{1}{2}+\frac{15i\sqrt{3}}{2}, -3+7i\sqrt{3}, -2+7i\sqrt{3}, -1+7i\sqrt{3}, -\frac{9}{2}+\frac{13i\sqrt{3}}{2}, -\frac{7}{2}+\frac{13i\sqrt{3}}{2}, -\frac{5}{2}+\frac{13i\sqrt{3}}{2}, -\frac{3}{2}+\frac{13i\sqrt{3}}{2}, -\frac{1}{2}+\frac{13i\sqrt{3}}{2}, -5+6i\sqrt{3}, -4+6i\sqrt{3}, -3+6i\sqrt{3}, -2+6i\sqrt{3}, -1+6i\sqrt{3}, -\frac{13}{2}+\frac{11i\sqrt{3}}{2}, -\frac{11}{2}+\frac{11i\sqrt{3}}{2}, -\frac{9}{2}+\frac{11i\sqrt{3}}{2}, -\frac{7}{2}+\frac{11i\sqrt{3}}{2}, -\frac{5}{2}+\frac{11i\sqrt{3}}{2}, -\frac{3}{2}+\frac{11i\sqrt{3}}{2}, -\frac{1}{2}+\frac{11i\sqrt{3}}{2}, -6+5i\sqrt{3}, -5+5i\sqrt{3}, -4+5i\sqrt{3}, -3+5i\sqrt{3}, -2+5i\sqrt{3}, -1+5i\sqrt{3}, -\frac{13}{2}+\frac{9i\sqrt{3}}{2}, -\frac{11}{2}+\frac{9i\sqrt{3}}{2}, -\frac{9}{2}+\frac{9i\sqrt{3}}{2}, -\frac{7}{2}+\frac{9i\sqrt{3}}{2}, -\frac{5}{2}+\frac{9i\sqrt{3}}{2}, -\frac{3}{2}+\frac{9i\sqrt{3}}{2}, -\frac{1}{2}+\frac{9i\sqrt{3}}{2}, -7+4i\sqrt{3}, -6+4i\sqrt{3}, -5+4i\sqrt{3}, -4+4i\sqrt{3}, -3+4i\sqrt{3}, -2+4i\sqrt{3}, -1+4i\sqrt{3}, -\frac{13}{2}+\frac{7i\sqrt{3}}{2}, -\frac{11}{2}+\frac{7i\sqrt{3}}{2}, -\frac{9}{2}+\frac{7i\sqrt{3}}{2}, -\frac{7}{2}+\frac{7i\sqrt{3}}{2}, -\frac{5}{2}+\frac{7i\sqrt{3}}{2}, -\frac{3}{2}+\frac{7i\sqrt{3}}{2}, -\frac{1}{2}+\frac{7i\sqrt{3}}{2}, -7+3i\sqrt{3}, -6+3i\sqrt{3}, -5+3i\sqrt{3}, -4+3i\sqrt{3}, -3+3i\sqrt{3}, -2+3i\sqrt{3}, -1+3i\sqrt{3}, -\frac{13}{2}+\frac{5i\sqrt{3}}{2}, -\frac{11}{2}+\frac{5i\sqrt{3}}{2}, -\frac{9}{2}+\frac{5i\sqrt{3}}{2}, -\frac{7}{2}+\frac{5i\sqrt{3}}{2}, -\frac{5}{2}+\frac{5i\sqrt{3}}{2}, -\frac{3}{2}+\frac{5i\sqrt{3}}{2}, -\frac{1}{2}+\frac{5i\sqrt{3}}{2}, -6+2i\sqrt{3}, -5+2i\sqrt{3}, -4+2i\sqrt{3}, -3+2i\sqrt{3}, -2+2i\sqrt{3}, -1+2i\sqrt{3}, -\frac{13}{2}+\frac{3i\sqrt{3}}{2}, -\frac{11}{2}+\frac{3i\sqrt{3}}{2}, -\frac{9}{2}+\frac{3i\sqrt{3}}{2}, -\frac{7}{2}+\frac{3i\sqrt{3}}{2}, -\frac{5}{2}+\frac{3i\sqrt{3}}{2}, -\frac{3}{2}+\frac{3i\sqrt{3}}{2}, -5+i\sqrt{3}, -4+i\sqrt{3}, -3+i\sqrt{3}, -2+i\sqrt{3}, -1+i\sqrt{3}, -\frac{9}{2}+\frac{i\sqrt{3}}{2}, -\frac{7}{2}+\frac{i\sqrt{3}}{2}, -\frac{5}{2}+\frac{i\sqrt{3}}{2}, -\frac{3}{2}+\frac{i\sqrt{3}}{2}, -\frac{1}{2}+\frac{i\sqrt{3}}{2}\}$.

$\mathcal{L}(t,\alpha,\beta)$		
Discriminant	$N_{k(\alpha)/k}(\beta)$	β
$D=1$ and $\lvert t\rvert \geq 6$ and $t \in \{-3+5i, -5+3i, -4+4i, -5+2i, -5+i, -4+3i, -4+2i, -4+i\}$	$\frac{1+i}{2}(2t+1) - \frac{5}{2}(1-i)$	$\{-(\alpha+b), (1-b)\alpha+1, b\alpha-1+b\}$

$\mathcal{L}(t,\alpha,\beta)$		
$D=1$ and $\lvert t\rvert \geq 4$ and $t\in\{-2+3i,-3+2i,$ $-2+2i,-3+i\}$	$\frac{1+i}{2}(2t+1)+\frac{5}{2}(1-i)$	$\{-(\alpha+1-b), b\alpha+1, (1-b)\alpha-b\}$
$D=1$ and $t\in\{-3+4i,-2+4i,$ $-1+4i,-3+3i,$ $-1+3i,-2+2i\}$	$\frac{1-3i}{2}(2t+1)-\frac{9+7i}{2}$	$\{\alpha+2-b,(1-b)\alpha-1,-((2-b)\alpha+1-b)\}$
$D=1$ and $t\in\{-1+5i,-2+4i,$ $-1+4i,-2+3i\}$	$-(2+i)(2t+1)-$ $7+11i$	$\{\alpha+1-2b,-(2b\alpha+1),(2b-1)\alpha+2b\}$
$D=1$ and $t=-1+5i$ $\lvert t\rvert=\sqrt{26}$	$1+10i$ $3-8i$ $9-4i$	$\{-(\alpha-2b),(1+2b)\alpha+1,-(2b\alpha+1+2b)\}$ $\{\alpha+1-4b,-(4b\alpha+1),(4b-1)\alpha+4b\}$ $\{-(\alpha^2+(4-5b)\alpha-1-b),(4-4b)\alpha^2+(2-5b)\alpha-1,$ $(1+b)\alpha^2+(6-3b)\alpha+4-4b\}$
$D=1$ and $t=-2+4i$ $\lvert t\rvert=2\sqrt{5}$	$6-6i$ $6-i$ $4-5i$	$\{2\alpha+1-b,-((1+b)\alpha+2),-((1-b)\alpha-1-b)\}$ $\{\alpha+2-3b,(1-3b)\alpha-1,-((2-3b)\alpha+1-3b)\}$ $\{\alpha^2+(2-3b)\alpha-b,-((1-2b)\alpha^2-3b\alpha-1),$ $-(b\alpha^2+(2-b)\alpha+1-2b)\}$
$D=1$ and $t=-3+3i$ $\lvert t\rvert=3\sqrt{2}$	$5+6i$	$\{\alpha+3-2b,(2-2b)\alpha-1,-((3-2b)\alpha+2-2b)\}$
$D=1$ and $t=-1+4i$ $\lvert t\rvert=\sqrt{17}$	$1-8i$	$\{\alpha^2+(3-2b)\alpha-b,(b-2)\alpha^2+(2b-1)\alpha+1,$ $-(b\alpha^2+3\alpha+2-b)\}$ or $\{-(\alpha^2+4\alpha-b),(3+b)\alpha^2+2\alpha-1,b\alpha^2+(4+2b)\alpha+3+b\}$
	$2-6i$	$\{2\alpha+1-b,-((1+b)\alpha+2),-((1-b)\alpha-1-b)\}$
	$3-4i$	$\{\alpha^2+(1-3b)\alpha+1,(1+3b)\alpha^2+(1+3b)\alpha+1,$ $\alpha^2+(1+3b)\alpha+1+3b\}$
	$2-3i$	$\{\alpha+1-3b,-(3b\alpha+1),-((1-3b)\alpha-3b)\}$
	$1-6i$	$\{\alpha-2b,-((1+2b)\alpha+1),2b\alpha+1+2b\}$
	$1+4i$	$\{-(\alpha^2+(1-3b)\alpha-b),-(2b\alpha^2+(1+3b)\alpha+1),$ $b\alpha^2+(1-b)\alpha-2b\}$
	$5-6i$	$\{\alpha^2+2\alpha-b,-((1+b)\alpha^2-1),-(b\alpha^2+(2+2b)\alpha+1+b)\}$
$D=1$ and $t=-2+3i$ $\lvert t\rvert=\sqrt{13}$	$6-2i$ $-3-4i$ $3+2i$ 3	$\{2\alpha+1-b,-((1+b)\alpha+2),-((1-b)\alpha-1-b)\}$ $\{\alpha^2+(3-2b)\alpha+2,2b\alpha^2-(1-2b)\alpha+1,2\alpha^2+(1+2b)\alpha+2b\}$ $\{\alpha+2-2b,(1-2b)\alpha-1,-((2-2b)\alpha+1-2b)\}$ $\{\alpha^2+(2-2b)\alpha-b,(b-1)\alpha^2+2b\alpha+1,-(b\alpha^2+2\alpha+1-b)\}$

$\mathcal{L}(t,\alpha,\beta)$		
$D=1$ and $t=-1+3i$ $\lvert t\rvert=\sqrt{10}$	$1-6i$	$\{-((1+2b)\alpha^2+(3+b)\alpha+1),(1-b)\alpha^2+(1-3b)\alpha-1-2b,$ $-(\alpha^2-(1+b)\alpha-1+b)\},$ $\{-(\alpha^2-b\alpha-1+b),-(2b\alpha^2+(2+b)\alpha+1),(1-b)\alpha^2+(2-3b)\alpha-2b\},$ $\{-(\alpha^2+(2+b)\alpha+1-b),2b\alpha^2+b\alpha-1,-((1-b)\alpha^2-3b\alpha-2b)\},$ $\{-(\alpha^2+(2-5b)\alpha-2-b),(3-4b)\alpha^2-5b\alpha-1,(2+b)\alpha^2+(6-3b)\alpha+3-4b\},$ $\{-(\alpha^2+(3+b)\alpha+2),b\alpha^2+(1+b)\alpha-1,-(2\alpha^2+(1-b)\alpha-b)\}$ or $\{-(\alpha^2+(6+2b)\alpha+5),2b\alpha^2+(4+2b)\alpha-1,-(5\alpha^2+(4-2b)\alpha-2b)\}$
	$2-2i$	$\{2\alpha+1-b,-((1+b)\alpha+2),-((1-b)\alpha-1-b)\}$ or $\{-((2-2b)\alpha-1-b),(3-b)\alpha+2-2b,-((1+b)\alpha+3-b)\}$
	$2-5i$	$\{3\alpha+2-b,-((1+b)\alpha+3),-((2-b)\alpha-1-b)\},$ $\{-(\alpha^2-(1+2b)\alpha-1+b),-((1+3b)\alpha^2+(3+2b)\alpha+1),$ $(1-b)\alpha^2+(1-4b)\alpha-1-3b\},$ $\{-(\alpha^2+3\alpha-b),(2+b)\alpha^2+\alpha-1,b\alpha^2+(3+2b)\alpha+2+b\},$ $\{-(\alpha^2+3\alpha+7-b),-((5-b)\alpha^2-\alpha+1),-((7-b)\alpha^2+(11-2b)\alpha+5-b)\},$ $\{\alpha^2+(3-2b)\alpha-b,(b-2)\alpha^2+(2b-1)\alpha+1,-(b\alpha^2+3\alpha+2-b)\},$ $\{2\alpha^2+(2-5b)\alpha-2-2b,-((2-3b)\alpha^2-(2+5b)\alpha-2),$ $-((2+2b)\alpha^2+(6-b)\alpha+2-3b)\}$ or $\{-(2\alpha^2+(3-4b)\alpha-2-b),(3-3b)\alpha^2-(1+4b)\alpha-2,$ $(2+b)\alpha^2+(7-2b)\alpha+3-3b\}$
	$4+i$	$\{\alpha+2-b,(1-b)\alpha-1,-((2-b)\alpha+1-b)\},$ $\{-((4+5b)\alpha+2-2b),(2+7b)\alpha+4+5b,(2-2b)\alpha-2-7b\},$ $\{\alpha^2+b,(1+b)\alpha^2+2\alpha+1,b\alpha^2+2b\alpha+1+b\},$ $\{\alpha^2+(1-2b)\alpha-1+b,-((1-3b)\alpha^2-(1+2b)\alpha-1),$ $-((1-b)\alpha^2+(3-4b)\alpha+1-3b)\},$ $\{\alpha^2+(1-2b)\alpha+1,(1+2b)\alpha^2+(1+2b)\alpha+1,\alpha^2+(1+2b)\alpha+1+2b\},$ $\{-(2\alpha^2+(2-4b)\alpha-2-b),(2-3b)\alpha^2-(2+4b)\alpha-2,$ $(2+b)\alpha^2+(6-2b)\alpha+2-3b\}$ or $\{-(2\alpha^2+(4-5b)\alpha-2-2b),(4-3b)\alpha^2-5b\alpha-2,$ $(2+2b)\alpha^2+(8-b)\alpha+4-3b\}$
	$3-2i$	$\{-(\alpha-2b),(1+2b)\alpha+1,-(2b\alpha+1+2b)\},$ $\{\alpha^2-(2+b)\alpha+b,(3+2b)\alpha^2+(4+b)\alpha+1,b\alpha^2+(2+3b)\alpha+3+2b\}$ or $\{-(3\alpha^2+(3-7b)\alpha-3-b),(3-6b)\alpha^2-(3+7b)\alpha-3,$ $(3+b)\alpha^2+(9-5b)\alpha+3-6b\}$
$D=1$ and $t=-2+2i$ $\lvert t\rvert=2\sqrt{2}$	$3-4i$	$\{\alpha^2+2\alpha-1-b,-((2+b)\alpha^2-1),-((1+b)\alpha^2+(4+2b)\alpha+2+b)\}$
$D=1$ and $t=-1+2i$ $\lvert t\rvert=\sqrt{5}$	$-1+4i$	$\{3\alpha+2-b,-((1+b)\alpha+3),-((2-b)\alpha-1-b)\},$ $\{\alpha^2+b,(1+b)\alpha^2+2\alpha+1,b\alpha^2+2b\alpha+1+b\}$ or $\{\alpha^2-(1+b)\alpha+b,(2+2b)\alpha^2+(3+b)\alpha+1,$ $b\alpha^2+(1+3b)\alpha+2+2b\}$
	$2+3i$	$\{-(\alpha-1-b),(2+b)\alpha+1,-((1+b)\alpha+2+b)\}$ or $\{-(2\alpha^2+(4-3b)\alpha-b),(2-2b)\alpha^2-3b\alpha-2,$ $b\alpha^2+(4-b)\alpha+2-2b\}$
	$2+2i$	$\{2\alpha+1-b,-((1+b)\alpha+2),-((1-b)\alpha-1-b)\}$

		$\mathcal{L}(t,\alpha,\beta)$
$D=2$ and $t\in\{-1+10i\sqrt{2}, -5+9i\sqrt{2}, -7+7i\sqrt{2}, -7+4i\sqrt{2}, -5+2i\sqrt{2}\}$	$\frac{2+\sqrt{2}i}{2}(2t+1)+\frac{1}{2}(8-7\sqrt{2}i)$	$\{-(\alpha+1-b), b\alpha+1, (1-b)\alpha-b\}$
$D=2$ and $t\in M_3$ and $t\in\{-6+8i\sqrt{2}, -7+6i\sqrt{2}, -7+5i\sqrt{2}, -6+3i\sqrt{2}\}$	$\frac{2+\sqrt{2}i}{2}(2t+1)+\frac{1}{2}(8-7\sqrt{2}i)$	$\{-(\alpha+1-b), b\alpha+1, (1-b)\alpha-b\}$
$D=2$ and $t\in M_3$	$\frac{2-\sqrt{2}i}{2}(2t+1)-\frac{1}{2}(8+7\sqrt{2}i)$	$\{-(\alpha-b), (1+b)\alpha+1, -(b\alpha+1+b)\}$
$D=2$ and $t=-2+3i\sqrt{2}$ $\|t\|=\sqrt{22}$	$3-6i\sqrt{2}$	$\{\alpha^2+(2-2b)\alpha-b, (b-1)\alpha^2+2b\alpha+1, -(b\alpha^2+2\alpha+1-b)\}$
$D=2$ and $t=-1+3i\sqrt{2}$ $\|t\|=\sqrt{19}$	$3-4i\sqrt{2}$ $5+3i\sqrt{2}$	$\{\alpha+1-2b, -(2b\alpha+1), (2b-1)\alpha+2b\}$ $\{-(5\alpha-b), (5+b)\alpha+5, -(b\alpha+5+b)\}$
$D=2$ and $t=-2+2i\sqrt{2}$ $\|t\|=2\sqrt{3}$	$3+2i\sqrt{2}$	$\{\alpha+2-b, (1-b)\alpha-1, -((2-b)\alpha+1-b)\}$
$D=2$ and $t=-1+2i\sqrt{2}$ $\|t\|=3$	$1-4i\sqrt{2}$	$\{-(\alpha^2+(4+b)\alpha+3), b\alpha^2+(2+b)\alpha-1, -(3\alpha^2+(2-b)\alpha-b)\}$, $\{b\alpha^2+(3+3b)\alpha+4+b, (1-b)\alpha^2-(3+b)\alpha+b, (4+b)\alpha^2+(5-b)\alpha+1-b\}$ or $\{b\alpha^2+(2+4b)\alpha+3+2b, (1-b)\alpha^2-(2+2b)\alpha+b, (3+2b)\alpha^2+4\alpha+1-b\}$
	$4+2i\sqrt{2}$	$\{2\alpha+2-b, -(b\alpha+2), -((2-b)\alpha-b)\}$ or $\{-((2-b)\alpha^2+(2-3b)\alpha-2-b), (2-b)\alpha^2-(2+b)\alpha-2+b, (2+b)\alpha^2+(6-b)\alpha+2-b\}$
	$1-i\sqrt{2}$	$\{4\alpha+3-b, -((1+b)\alpha+4), -((3-b)\alpha-1-b)\}$
	$5-i\sqrt{2}$	$\{3\alpha+2-b, -((1+b)\alpha+3), -((2-b)\alpha-1-b)\}$
	$3-i\sqrt{2}$	$\{\alpha+2-b, (1-b)\alpha-1, -((2-b)\alpha+1-b)\}$, $\{2\alpha+1-b, -((1+b)\alpha+2), -((1-b)\alpha-1-b)\}$, $\{\alpha^2+2\alpha-b, -((1+b)\alpha^2-1), -(b\alpha^2+(2+2b)\alpha+1+b)\}$ or $\{\alpha^2+(1-b)\alpha+1, (1+b)\alpha^2+(1+b)\alpha+1, \alpha^2+(1+b)\alpha+1+b\}$
	$2i\sqrt{2}$	$\{-(2\alpha-b), (2+b)\alpha+2, -(b\alpha+2+b)\}$
	$1+2i\sqrt{2}$	$\{\alpha^2+\alpha+1, \alpha^2+\alpha+1, \alpha^2+\alpha+1\}$ or $\{-(\alpha^2+(2-b)\alpha+1), -(b\alpha^2+b\alpha+1), -(\alpha^2+b\alpha+b)\}$
	$5+i\sqrt{2}$	$\{-((1-b)\alpha^2-3b\alpha-2-b), (1-b)\alpha^2-(2+b)\alpha-1+b, (2+b)\alpha^2+(4-b)\alpha+1-b\}$

$\mathcal{L}(t,\alpha,\beta)$		
$D=3$ and $t\notin M_4$	$\frac{i\sqrt{3}}{2}(2t+1)-\frac{7}{2}$	$\{\alpha+1+b, b\alpha-1, -((1+b)\alpha+b)\}$
$D=3$ and $t\neq -\frac{1}{2}+\frac{i\sqrt{3}}{2}$	$\frac{i\sqrt{3}}{2}(2t+1)+\frac{7}{2}$ $\frac{2t+1}{2}-\frac{3i\sqrt{3}}{2}$	$\{\alpha-b, -((1+b)\alpha+1), b\alpha+1+b\}$ $\{-(\alpha+1-b), b\alpha+1, (1-b)\alpha-b\}$
$D=3$ and $\lvert t\rvert>2$	$\frac{2t+1}{2}+\frac{3i\sqrt{3}}{2}$	$\{-(\alpha+b), (1-b)\alpha+1, b\alpha-1+b\}$
$D=3$ and $t\in\{-\frac{1}{2}+\frac{7i\sqrt{3}}{2}, -2+3i\sqrt{3},$ $-1+3i\sqrt{3}, -\frac{5}{2}+\frac{5i\sqrt{3}}{2},$ $-\frac{3}{2}+\frac{5i\sqrt{3}}{2}, -\frac{1}{2}+\frac{5i\sqrt{3}}{2},$ $-2+2i\sqrt{3}, -1+2i\sqrt{3}\}$	$-\frac{3-i\sqrt{3}}{2}(2t+1)$ $+\frac{11+9i\sqrt{3}}{2}$ $-\frac{3+i\sqrt{3}}{2}(2t+1)$ $-\frac{11-9i\sqrt{3}}{2}$ $\frac{2t+1}{2}-\frac{3i\sqrt{3}}{2}$	$\{-(\alpha+1-2b), 2b\alpha+1, -((2b-1)\alpha+2b)\}$ $\{\alpha+2-2b, (1-2b)\alpha-1, -((2-2b)\alpha+1-2b)\}$ $\{(\alpha+1-b)^2, (b\alpha+1)^2, ((1-b)\alpha-b)^2\}$
$D=3$ and $t=-1+3i\sqrt{3}$ $\lvert t\rvert=2\sqrt{7}$	$\frac{19}{2}+\frac{5i\sqrt{3}}{2}$	$\{-(\alpha^2+(4-4b)\alpha-b),$ $-((3b-3)\alpha^2+(4b-2)\alpha+1),$ $b\alpha^2-(2b-4)\alpha+3-3b\}$
$D=3$ and $t=-\frac{5}{2}+\frac{5i\sqrt{3}}{2}$ $\lvert t\rvert=5$	$4-5i\sqrt{3}$	$\{2\alpha^2+(4-b)\alpha-b, -(2\alpha^2-b\alpha-2),$ $-(b\alpha^2+(4+b)\alpha+2)\}$
$D=3$ and $t=-3+2i\sqrt{3}$ $\lvert t\rvert=\sqrt{21}$	$\frac{17}{2}-\frac{i\sqrt{3}}{2}$	$\{\alpha+5-3b, (4-3b)\alpha-1, -((5-3b)\alpha+4-3b)\}$
$D=3$ and $t=-\frac{3}{2}+\frac{5i\sqrt{3}}{2}$ $\lvert t\rvert=\sqrt{21}$	$1-4i\sqrt{3}$ $7-3i\sqrt{3}$ $8-i\sqrt{3}$ $2+3i\sqrt{3}$ $5+4i\sqrt{3}$	$\{\alpha+4-4b, (3-4b)\alpha-1, -((4-4b)\alpha+3-4b)\}$ $\{\alpha+3-3b, (2-3b)\alpha-1, -((3-3b)\alpha+2-3b)\}$ $\{\alpha+3-4b, (2-4b)\alpha-1, -((3-4b)\alpha+2-4b)\}$ $\{-(\alpha^2+(4-3b)\alpha-b), (3-2b)\alpha^2+(2-3b)\alpha-1,$ $b\alpha^2+(4-b)\alpha+3-2b\}$ $\{-(\alpha^2+(4-4b)\alpha+1), (2-4b)\alpha^2+(2-4b)\alpha-1,$ $-(\alpha^2-(2-4b)\alpha-2+4b)\}$
$D=3$ and $t=-\frac{1}{2}+\frac{5i\sqrt{3}}{2}$ $\lvert t\rvert=\sqrt{19}$	-4 $4\sqrt{3}i$ $4-3\sqrt{3}i$ $4+3\sqrt{3}i$ -5 8	$\{\alpha^2+(3-4b)\alpha+1-2b, (2b-1)\alpha^2+(4b-1)\alpha+1,$ $-((2b-1)\alpha^2+\alpha+1-2b)\}$ $\{\alpha+2-3b, (1-3b)\alpha-1, -((2-3b)\alpha+1-3b)\}$ $\{\alpha+2-4b, (1-4b)\alpha-1, -((2-4b)\alpha+1-4b)\}$ $\{-(\alpha+3-4b), -((2-4b)\alpha-1), (3-4b)\alpha+2-4b\}$ $\{\alpha^2+(4-4b)\alpha-b, (3b-3)\alpha^2+(4b-2)\alpha+1,$ $-(b\alpha^2-(2b-4)\alpha+3-3b)\}$ $\{\alpha^2+(4-6b)\alpha-1-b, (5b-4)\alpha^2+(6b-2)\alpha+1,$ $-((b+1)\alpha^2-(4b-6)\alpha+4-5b)\}$
$D=3$ and $t=-2+2i\sqrt{3}$ $\lvert t\rvert=4$	$6+i\sqrt{3}$ $\frac{11}{2}+\frac{3i\sqrt{3}}{2}$ $\frac{7}{2}-\frac{3i\sqrt{3}}{2}$ $\frac{5}{2}-\frac{3i\sqrt{3}}{2}$	$\{\alpha+3-2b, (2-2b)\alpha-1, -((3-2b)\alpha+2-2b)\}$ $\{\alpha+3-3b, (2-3b)\alpha-1, -((3-3b)\alpha+2-3b)\}$ $\{\alpha+4-3b, (3-3b)\alpha-1, -((4-3b)\alpha+3-3b)\}$ $\{\alpha^2+(3-2b)\alpha-b, (b-2)\alpha^2+(2b-1)\alpha+1,$ $-(b\alpha^2+3\alpha+2-b)\}$

$\mathcal{L}(t,\alpha,\beta)$		
$D=3$ and $t=-1+2i\sqrt{3}$ $\lvert t\rvert=\sqrt{13}$	$1-4i\sqrt{3}$	$\{\alpha^2+(2-2b)\alpha+1, 2b\alpha^2+2b\alpha+1, \alpha^2+2b\alpha+2b\}$
	$-\frac{1}{2}-\frac{7i\sqrt{3}}{2}$	$\{\alpha^2+(2-b)\alpha+1, b\alpha^2+b\alpha+1, \alpha^2+b\alpha+b\}$
	$\frac{9}{2}+\frac{3i\sqrt{3}}{2}$	$\{(1-2b)\alpha-1-b, -((2-b)\alpha+1-2b), (1+b)\alpha+2-b\}$
	$\frac{9}{2}-\frac{3i\sqrt{3}}{2}$	$\{(1-2b)\alpha^2-(3+3b)\alpha-1-b, 3\alpha^2+(5-b)\alpha+1-2b, -((1+b)\alpha^2-(1+b)\alpha-3)\}$
	$5-2i\sqrt{3}$	$\{\alpha+3-2b, (2-2b)\alpha-1, -((3-2b)\alpha+2-2b)\}$
	$5+2i\sqrt{3}$	$\{\alpha^2+\alpha+1, \alpha^2+\alpha+1, \alpha^2+\alpha+1\}$
	$\frac{7}{2}-\frac{i\sqrt{3}}{2}$	$\{\alpha+2-3b, (1-3b)\alpha-1, -((2-3b)\alpha+1-3b)\}$
	$\frac{1}{2}+\frac{3i\sqrt{3}}{2}$	$\{-(\alpha+3-3b), -((2-3b)\alpha-1), (3-3b)\alpha+2-3b\}$
	$-\frac{13}{2}+\frac{3i\sqrt{3}}{2}$	$\{\alpha^2+(1+b)\alpha+1, (1-b)\alpha^2+(1-b)\alpha+1, \alpha^2+(1-b)\alpha+1-b\}$ or $\{\alpha^2+(3-2b)\alpha+4-b, (2+b)\alpha^2-(1-2b)\alpha+1, (4-b)\alpha^2+5\alpha+2+b\}$
	$\frac{11}{2}+\frac{3i\sqrt{3}}{2}$	$\{\alpha^2+(1+b)\alpha+1-b, (1-2b)\alpha^2+(1-b)\alpha+1, (1-b)\alpha^2+(1-3b)\alpha+1-2b\}$
	$\frac{11}{2}-\frac{3i\sqrt{3}}{2}$	$\{\alpha^2+(3+3b)\alpha+2-2b, -(5b\alpha^2+(1+3b)\alpha-1), (2-2b)\alpha^2+(1-7b)\alpha-5b\}$
	$\frac{5}{2}+\frac{3i\sqrt{3}}{2}$	$\{\alpha^2+(4-5b)\alpha-1-b, -((4-4b)\alpha^2+(2-5b)\alpha-1), -((1+b)\alpha^2+(6-3b)\alpha+4-4b)\}$
	$\frac{11}{2}+\frac{5i\sqrt{3}}{2}$	$\{(\alpha-b)^2, ((1+b)\alpha+1)^2, (b\alpha+1+b)^2\}$ or $\{-((1+b)\alpha^2+(7-3b)\alpha+6-5b), b\alpha^2+(5-5b)\alpha-1-b, -((6-5b)\alpha^2+(5-7b)\alpha-b)\}$
$D=3$ and $t=-\frac{5}{2}+\frac{3i\sqrt{3}}{2}$ $\lvert t\rvert=\sqrt{13}$	$2+3i\sqrt{3}$	$\{\alpha+3-2b, (2-2b)\alpha-1, -((3-2b)\alpha+2-2b)\}$
	$5+2i\sqrt{3}$	$\{\alpha+2-2b, (1-2b)\alpha-1, -((2-2b)\alpha+1-2b)\}$
	$4+i\sqrt{3}$	$\{\alpha+4-2b, (3-2b)\alpha-1, -((4-2b)\alpha+3-2b)\}$
	4	$\{(\alpha+1-b)^2, (b\alpha+1)^2, ((1-b)\alpha-b)^2\}$
$D=3$ and $t=-\frac{3}{2}+\frac{3i\sqrt{3}}{2}$ $\lvert t\rvert=3$	$2-3i\sqrt{3}$	$\{\alpha^2+(3-b)\alpha+2, b\alpha^2-(1-b)\alpha+1, 2\alpha^2+(1+b)\alpha+b\}$ or $\{2\alpha^2+(3-2b)\alpha+1-b, b\alpha^2+(1+2b)\alpha+2, (1-b)\alpha^2-\alpha+b\}$
	$1-3i\sqrt{3}$	$\{\alpha^2+\alpha+1, \alpha^2+\alpha+1, \alpha^2+\alpha+1\}$ or $\{\alpha^2+(3-b)\alpha+2+b, 2b\alpha^2-(1-b)\alpha+1, (2+b)\alpha^2+(1+3b)\alpha+2b\}$
	$2+i\sqrt{3}$	$\{\alpha+2-2b, (1-2b)\alpha-1, -((2-2b)\alpha+1-2b)\}$
	$3i\sqrt{3}$	$\{(1-2b)\alpha-1-b, -((2-b)\alpha+1-2b), (1+b)\alpha+2-b\}$ or $\{(4-2b)\alpha-1-b, -((5-b)\alpha+4-2b), (1+b)\alpha+5-b\}$
	$4-i\sqrt{3}$	$\{\alpha+1-2b, -(2b\alpha+1), (2b-1)\alpha+2b\}$, $\{\alpha^2+(2-b)\alpha+1, b\alpha^2+b\alpha+1, \alpha^2+b\alpha+b\}$ or $\{\alpha^2+(3-b)\alpha+1, -((1-b)\alpha^2+(1-b)\alpha-1), \alpha^2-(1-b)\alpha-1+b\}$
	$-2+2i\sqrt{3}$	$\{(\alpha-b)^2, ((1+b)\alpha+1)^2, (b\alpha+1+b)^2\}$
$D=3$ and $t=-2+i\sqrt{3}$ $\lvert t\rvert=\sqrt{7}$	$\frac{1}{2}+\frac{3i\sqrt{3}}{2}$	$\{\alpha+3-b, (2-b)\alpha-1, -((3-b)\alpha+2-b)\}$
	$\frac{3}{2}+\frac{3i\sqrt{3}}{2}$	$\{(\alpha+1-b)^2, (b\alpha+1)^2, ((1-b)\alpha-b)^2\}$
$D=3$ and $t=-1+i\sqrt{3}$ $\lvert t\rvert=2$	$\frac{7}{2}-\frac{i\sqrt{3}}{2}$	$\{\alpha+3-b, (2-b)\alpha-1, -((3-b)\alpha+2-b)\}$
	$\frac{5}{2}+\frac{3i\sqrt{3}}{2}$	$\{\alpha^2+(2-b)\alpha+1, b\alpha^2+b\alpha+1, \alpha^2+b\alpha+b\}$
$D=3$ and $t=-\frac{3}{2}+\frac{i\sqrt{3}}{2}$ $\lvert t\rvert=\sqrt{3}$	$1+i\sqrt{3}$	$\{-(\alpha+3-b), -((2-b)\alpha-1), (3-b)\alpha+2-b\}$

$\mathcal{L}(t,\alpha,\beta)$		
$D=5$ and $t=-1+2i\sqrt{5}$ $\lvert t\rvert=\sqrt{21}$	$1-4i\sqrt{5}$ $4-3i\sqrt{5}$	$\{\alpha-1,-(2\alpha+1),\alpha+2\}$ or $\{\alpha-b,-((1+b)\alpha+1),b\alpha+1+b\}$ $\{\alpha+1-b,-(b\alpha+1),-((1-b)\alpha-b)\}$
$D=5$ and $t=-1+i\sqrt{5}$ $\lvert t\rvert=\sqrt{6}$	t^2+t+7	$\{\alpha^2+\alpha+1,\alpha^2+\alpha+1,\alpha^2+\alpha+1\}$
$D=6$ and $t=-1+i\sqrt{6}$ $\lvert t\rvert=\sqrt{7}$	$1-2i\sqrt{6}$ $1-i\sqrt{6}$	$\{\alpha-1,-(2\alpha+1),\alpha+2\}$ or $\{\alpha^2+2\alpha+2,\alpha^2+1,2\alpha^2+2\alpha+1\}$ $\{\alpha^2+\alpha+1,\alpha^2+\alpha+1,\alpha^2+\alpha+1\}$
$D=7$ and $t\in\{-1+3i\sqrt{7},-\frac{7}{2}+\frac{5i\sqrt{7}}{2},$ $-\frac{5}{2}+\frac{5i\sqrt{7}}{2},-\frac{3}{2}+\frac{5i\sqrt{7}}{2},$ $-\frac{1}{2}+\frac{5i\sqrt{7}}{2},-4+2i\sqrt{7},$ $-3+2i\sqrt{7},-2+2i\sqrt{7},$ $-1+2i\sqrt{7},-\frac{9}{2}+\frac{3i\sqrt{7}}{2},$ $-\frac{7}{2}+\frac{3i\sqrt{7}}{2},-\frac{5}{2}+\frac{3i\sqrt{7}}{2},$ $-\frac{3}{2}+\frac{3i\sqrt{7}}{2},-\frac{1}{2}+\frac{3i\sqrt{7}}{2},$ $-3+i\sqrt{7},-2+i\sqrt{7}\}$	$\frac{1+i\sqrt{7}}{2}(2t+1)+$ $\frac{13-i\sqrt{7}}{2}$	$\{-(\alpha+2-b),(1-b)\alpha-1,(2-b)\alpha+1-b\}$
$D=7$ and $t\in\{-\frac{1}{2}+\frac{5i\sqrt{7}}{2},-2+2i\sqrt{7},$ $-1+2i\sqrt{7},-\frac{5}{2}+\frac{3i\sqrt{7}}{2},$ $-\frac{3}{2}+\frac{3i\sqrt{7}}{2},-\frac{1}{2}+\frac{3i\sqrt{7}}{2},$ $-2+i\sqrt{7},-1+i\sqrt{7}\}$	$\frac{1-i\sqrt{7}}{2}(2t+1)-$ $\frac{13+i\sqrt{7}}{2}$	$\{-(\alpha-b),(1+b)\alpha+1,-(b\alpha+1+b)\}$
$D=7$ and $\lvert t\rvert\geq 2$	$2t+1-i2\sqrt{7}$	$\{-(\alpha+1-b),b\alpha+1,(1-b)\alpha-b\}$
$D=7$ and $t=-\frac{3}{2}+\frac{3i\sqrt{7}}{2}$ $\lvert t\rvert=3\sqrt{2}$	$6-i\sqrt{7}$ $5-2i\sqrt{7}$ $1+2i\sqrt{7}$	$\{\alpha+2-2b,(1-2b)\alpha-1,-((2-2b)\alpha+1-2b)\}$ $\{\alpha+3-2b,(2-2b)\alpha-1,-((3-2b)\alpha+2-2b)\}$ $\{-(\alpha^2+(3-2b)\alpha+1-b),$ $(1-b)\alpha^2+(1-2b)\alpha-1,$ $-((1-b)\alpha^2-\alpha-1+b)\}$
$D=7$ and $t=-\frac{1}{2}+\frac{3i\sqrt{7}}{2}$ $\lvert t\rvert=4$	$1-2i\sqrt{7}$ $1+2i\sqrt{7}$ $\frac{7}{2}+\frac{5i\sqrt{7}}{2}$ $\frac{7}{2}-\frac{5i\sqrt{7}}{2}$ -3 -5 -7	$\{\alpha+1-2b,-(2b\alpha+1),(2b-1)\alpha+2b\}$ $\{-(\alpha+2-2b),-((1-2b)\alpha-1),(2-2b)\alpha+1-2b\}$ $\{-(2\alpha+1-b),(1+b)\alpha+2,(1-b)\alpha-1-b\}$ $\{2\alpha+2-b,-(b\alpha+2),-((2-b)\alpha-b)\}$ $\{\alpha^2+(2-2b)\alpha-b,(b-1)\alpha^2+2b\alpha+1,$ $-(b\alpha^2+2\alpha+1-b)\}$ $\{\alpha^2+(3-2b)\alpha-b,(b-2)\alpha^2+(2b-1)\alpha+1,$ $-(b\alpha^2+3\alpha+2-b)\}$ $\{\alpha^2+(4-2b)\alpha-b,(b-3)\alpha^2+(2b-2)\alpha+1,$ $-(b\alpha^2+4\alpha+3-b)\}$ or $\{(\alpha+1-b)^2,(b\alpha+1)^2$ or $((1-b)\alpha-b)^2\}$
$D=7$ and $t=-2+i\sqrt{7}$ $\lvert t\rvert=\sqrt{11}$	$4+i\sqrt{7}$ 5	$\{\alpha+3-b,(2-b)\alpha-1,-((3-b)\alpha+2-b)\}$ $\{\alpha^2+(3-b)\alpha+1,-((1-b)\alpha^2+(1-b)\alpha-1),$ $\alpha^2-(1-b)\alpha-1+b\}$

$\mathcal{L}(t,\alpha,\beta)$		
$D=7$ and $t=-1+i\sqrt{7}$ $\lvert t\rvert=2\sqrt{2}$	$1-2i\sqrt{7}$	$\{(5-3b)\alpha-2-b,-((7-2b)\alpha+5-3b),$ $(2+b)\alpha+7-2b\},$ $\{-(\alpha^2+\alpha+1-b),-((1-b)\alpha^2+\alpha+1),$ $-((1-b)\alpha^2+(1-2b)\alpha+1-b)\},$ $\{-(\alpha^2+(3-3b)\alpha-1-b),(3-2b)\alpha^2+(1-3b)\alpha-1,$ $(1+b)\alpha^2+(5-b)\alpha+3-2b\},$ $\{-(\alpha^2+(4-3b)\alpha-2-b),(5-2b)\alpha^2+(2-3b)\alpha-1,$ $(2+b)\alpha^2+(8-b)\alpha+5-2b\},$ $\{3\alpha^2+(4-2b)\alpha+1-b,b\alpha^2+(2+2b)\alpha+3,$ $(1-b)\alpha^2-2\alpha+b\},$ $\{(2-b)\alpha^2+(3-3b)\alpha-1-b,$ $-((2-b)\alpha^2-(1+b)\alpha-2+b),$ $-((1+b)\alpha^2+(5-b)\alpha+2-b)\}$ or $\{-((11-5b)\alpha^2-(2+3b)\alpha-6+4b),$ $-((7+2b)\alpha^2+(24-7b)\alpha+11-5b),$ $(6-4b)\alpha^2+(10-11b)\alpha-7-2b\}$
	$1+2i\sqrt{7}$	$\{3\alpha+3-b,-(b\alpha+3),-((3-b)\alpha-b)\},$ $\{3\alpha-b,-((3+b)\alpha+3),b\alpha+3+b\},$ $\{(1-2b)\alpha-1-b,-((2-b)\alpha+1-2b),$ $(1+b)\alpha+2-b\},$ $\{3\alpha+6-4b,(3-4b)\alpha-3,-((6-4b)\alpha+3-4b)\},$ $\{-(\alpha^2+2\alpha-b),(1+b)\alpha^2-1,b\alpha^2+(2+2b)\alpha+1+b\},$ $\{-(\alpha^2+2\alpha+3+b),-((2+b)\alpha^2+1),$ $-((3+b)\alpha^2+(4+2b)\alpha+2+b)\}$ or $\{-(b\alpha^2+(1+3b)\alpha+2+b),$ $-((1-b)\alpha^2-(1+b)\alpha+b),$ $-((2+b)\alpha^2+(3-b)\alpha+1-b)\}$
	$\frac{5}{2}-\frac{i\sqrt{7}}{2}$	$\{2\alpha+1-b,-((1+b)\alpha+2),-((1-b)\alpha-1-b)\}$ or $\{-(2b\alpha+1+b),-((1-b)\alpha-2b),(1+b)\alpha+1-b\}$
	$\frac{5}{2}+\frac{i\sqrt{7}}{2}$	$\{2\alpha+2-b,-(b\alpha+2),-((2-b)\alpha-b)\}$ or $\{-(2\alpha-b),(2+b)\alpha+2,-(b\alpha+2+b)\}$
	$-i\sqrt{7}$	$\{\alpha^2+\alpha+1,\alpha^2+\alpha+1,\alpha^2+\alpha+1\}$ or $\{4\alpha^2+(5-2b)\alpha+1-b,b\alpha^2+(3+2b)\alpha+4,$ $(1-b)\alpha^2-3\alpha+b\}$
$D=7$ and $t=-\frac{3}{2}+\frac{i\sqrt{7}}{2}$ $\lvert t\rvert=2$	$2+i\sqrt{7}$	$\{-(\alpha+3-b),-((2-b)\alpha-1),(3-b)\alpha+2-b\}$
$D=10$ and $t=-1+i\sqrt{10}$ $\lvert t\rvert=\sqrt{11}$	t^2+t+7	$\{\alpha^2+\alpha+1,\alpha^2+\alpha+1,\alpha^2+\alpha+1\}$
$D=11$ and $t\in\{-\frac{3}{2}+\frac{3i\sqrt{11}}{2},-\frac{1}{2}+\frac{3i\sqrt{11}}{2},$ $-3+i\sqrt{11},-2+i\sqrt{11},$ $-1+i\sqrt{11}\}$	$\frac{2+i\sqrt{11}}{2}(2t+1)+$ $\frac{1}{2}(19-2i\sqrt{11})$	$\{-(\alpha+2-b),-((1-b)\alpha-1),(2-b)\alpha+1-b\}$
$D=11$ and $t\in\{-\frac{1}{2}+\frac{3i\sqrt{11}}{2},-1+i\sqrt{11}\}$	$\frac{2-i\sqrt{11}}{2}(2t+1)-$ $\frac{1}{2}(19+2i\sqrt{11})$	$\{-(\alpha-b),(1+b)\alpha+1,-(b\alpha+1+b)\}$
$D=11$ and $t\in\{-\frac{1}{2}+\frac{5i\sqrt{11}}{2},-3+2i\sqrt{11},$ $-2+2i\sqrt{11},-1+2i\sqrt{11},$ $-\frac{7}{2}+\frac{3i\sqrt{11}}{2},-\frac{5}{2}+\frac{3i\sqrt{11}}{2},$ $-\frac{3}{2}+\frac{3i\sqrt{11}}{2},-\frac{1}{2}+\frac{3i\sqrt{11}}{2},$ $-3+i\sqrt{11},-2+i\sqrt{11},$ $-1+i\sqrt{11}\}$	$\frac{3}{2}(2t+1)-\frac{5i\sqrt{11}}{2}$	$\{-(\alpha+1-b),b\alpha+1,(1-b)\alpha-b\}$

$\mathcal{L}(t,\alpha,\beta)$		
$D=11$ and $t=-1+i\sqrt{11}$ $\lvert t\rvert=2\sqrt{3}$	$1-2i\sqrt{11}$ $-\frac{1}{2}-\frac{3i\sqrt{11}}{2}$ $4+i\sqrt{11}$ $-\frac{5}{2}-\frac{3i\sqrt{11}}{2}$	$\{2\alpha+1-b,-((1+b)\alpha+2),-((1-b)\alpha-1-b)\}$ or $\{2\alpha+4-3b,(2-3b)\alpha-2,-((4-3b)\alpha+2-3b)\}$ $\{\alpha^2+(2-b)\alpha+1,b\alpha^2+b\alpha+1,\alpha^2+b\alpha+b\}$ $\{\alpha^2+\alpha+1,\alpha^2+\alpha+1,\alpha^2+\alpha+1\}$ or $\{-(2\alpha^2+(2-2b)\alpha-1-b),$ $(1-b)\alpha^2-(2+2b)\alpha-2,$ $(1+b)\alpha^2+4\alpha+1-b\}$ $\{\alpha^2+(2-b)\alpha+2,(1+b)\alpha^2+b\alpha+1,$ $2\alpha^2+(2+b)\alpha+1+b\}$
$D=13$ and $t=-1+i\sqrt{13}$ $\lvert t\rvert=\sqrt{14}$	t^2+t+7	$\{\alpha^2+\alpha+1,\alpha^2+\alpha+1,\alpha^2+\alpha+1\}$
$D=15$ and $t\in\{-2+i\sqrt{15},-1+i\sqrt{15}\}$	$\frac{3+i\sqrt{15}}{2}(2t+1)+$ $\frac{1}{2}(25-3i\sqrt{15})$	$\{-(\alpha+2-b),-((1-b)\alpha-1),(2-b)\alpha+1-b\}$
$D=15$ and $t=-1+i\sqrt{15}$ $\lvert t\rvert=4$	$1+2i\sqrt{15}$	$\{-(\alpha-b),(1+b)\alpha+1,-(b\alpha+1+b)\}$
$D=15$ and $t\in\{-\frac{1}{2}+\frac{3i\sqrt{15}}{2},-2+i\sqrt{15},$ $-1+i\sqrt{15}\}$	$2(2t+1)-3i\sqrt{15}$	$\{-(\alpha+1-b),b\alpha+1,(1-b)\alpha-b\}$
$D=15$ and $t=-\frac{1}{2}+\frac{i\sqrt{15}}{2}$ $\lvert t\rvert=2$	3	$\{\alpha^2+\alpha+1,\alpha^2+\alpha+1,\alpha^2+\alpha+1\}$
$D=19$ and $t=-1+i\sqrt{19}$ $\lvert t\rvert=2\sqrt{5}$	$\frac{11}{2}-\frac{3i\sqrt{19}}{2}$ $\frac{5}{2}-\frac{3i\sqrt{19}}{2}$	$\{\alpha+2-b,(1-b)\alpha-1,-((2-b)\alpha+1-b)\}$ $\{\alpha+1-b,-(b\alpha+1),-((1-b)\alpha-b)\}$
$D=19$ and $t=-\frac{1}{2}+\frac{i\sqrt{19}}{2}$ $\lvert t\rvert=\sqrt{5}$	-3 2 4	$\{2\alpha+3-b,(1-b)\alpha-2,-((3-b)\alpha+1-b)\}$ $\{\alpha^2+\alpha+1,\alpha^2+\alpha+1,\alpha^2+\alpha+1\}$ $\{(\alpha^2+\alpha+1)^2,(\alpha^2+\alpha+1)^2,(\alpha^2+\alpha+1)^2\}$
$D=23,31,35,39$ and $t=-\frac{3}{2}+\frac{i\sqrt{D}}{2}$	t^2+t+7	$\{\alpha^2+\alpha+1,\alpha^2+\alpha+1,\alpha^2+\alpha+1\}$
$D=23$ and $t=-\frac{1}{2}+\frac{i\sqrt{23}}{2}$ $\lvert t\rvert=\sqrt{6}$	$i\sqrt{23}$ $\frac{3}{2}+\frac{i\sqrt{23}}{2}$ $\frac{3}{2}-\frac{i\sqrt{23}}{2}$	$\{\alpha-1,-(2\alpha+1),\alpha+2\},$ $\{3\alpha+2-b,-((1+b)\alpha+3),-((2-b)\alpha-1-b)\}$ or $\{5\alpha+3-b,-((2+b)\alpha+5),-((3-b)\alpha-2-b)\}$ $\{2\alpha+1-b,-((1+b)\alpha+2),-((1-b)\alpha-1-b)\}$ $\{-(2\alpha+2-b),b\alpha+2,(2-b)\alpha-b\}$
$D=31$ and $t=-\frac{1}{2}+\frac{i\sqrt{31}}{2}$ $\lvert t\rvert=2\sqrt{2}$	$i\sqrt{31}$ $\frac{1}{2}-\frac{i\sqrt{31}}{2}$ $\frac{1}{2}+\frac{i\sqrt{31}}{2}$ -3	$\{\alpha-1,-(2\alpha+1),\alpha+2\},$ $\{-(3\alpha+2-b),(1+b)\alpha+3,(2-b)\alpha-1-b\}$ or $\{-(5\alpha+4-3b),(1+3b)\alpha+5,(4-3b)\alpha-1-3b\}$ $\{2\alpha+1-b,-((1+b)\alpha+2),-((1-b)\alpha-1-b)\}$ $\{-(2\alpha+2-b),b\alpha+2,(2-b)\alpha-b\}$ $\{\alpha^2+\alpha+2,2\alpha^2+\alpha+1,2\alpha^2+3\alpha+2\},$ $\{\alpha^2+2\alpha+2,\alpha^2+1,2\alpha^2+2\alpha+1\}$ or $\{(b+1)\alpha^2+(b+2)\alpha+2,\alpha^2+b\alpha+1+b,$ $2\alpha^2-(b-2)\alpha+1\}$
$D=35,39,43,47,51,55$ and $t=-\frac{1}{2}+\frac{i\sqrt{D}}{2}$	t^2+t+7	$\{\alpha^2+\alpha+1,\alpha^2+\alpha+1,\alpha^2+\alpha+1\}$
$D=35$ and $t=-\frac{1}{2}+\frac{i\sqrt{35}}{2}$ $\lvert t\rvert=3$	$i\sqrt{35}$ 4 -5	$\{\alpha-1,-(2\alpha+1),\alpha+2\},$ $\{-(2\alpha+1-b),(1+b)\alpha+2,(1-b)\alpha-1-b\}$ or $\{-(2\alpha+2-b),b\alpha+2,(2-b)\alpha-b\}$ $\{(\alpha^2+\alpha+1)^2,(\alpha^2+\alpha+1)^2,(\alpha^2+\alpha+1)^2\}$ $\{\alpha^2+\alpha+4,4\alpha^2+\alpha+1,4\alpha^2+7\alpha+4\}$ or $\{\alpha^2+2\alpha+2,\alpha^2+1,2\alpha^2+2\alpha+1\}$

Remark. In case $D = 2$ we have for all $t \in \{-1+10i\sqrt{2}, -5+9i\sqrt{2}, -7+7i\sqrt{2}, -7+4i\sqrt{2}, -5+2i\sqrt{2}\}$ that $|N_{k(\alpha)/k}(\alpha-1)| = |N_{k(\alpha)/k}(\alpha+1-b)| = |2t+1|$, but none of the conjugates of $\alpha+1-b$ is associated to any of the conjugates of $\alpha - 1$ since the quotient $\frac{N_{k(\alpha)/k}(\alpha+1-b)}{N_{k(\alpha)/k}(\alpha-1)}$ is not an element of $\mathbb{Z}_k$.

References

[1] Kirschenhofer, P., Lampl, C. and Thuswaldner, J. On a parameterized family of relative Thue equations. submitted.