Score-based Portfolio Choice

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Factor Investing: A new approach to stock investing

- In investment management recent academic literature has disrupted the industry
- Instead of old stock picking strategies, common risk factors are introduced with the purpose to explain the cross-section of stock returns
- For holding stocks exposed to common non-diversifiable risk factors, investors are compensated with excess returns
- Harvesting these excess returns has resulted in a large amount of literature about the topic

Factor Investing: Portfolio Choice

- The standard approach to factor investing estimates sensitivities to characteristics-sorted long-short portfolios
- Portfolios are then constructed based on the sensitivities to these factors
- Often these sensitivies are subject to a large amount of noise in individual factor loadings
- *"..., our evidence suggests that it is characteristics rather than factor loadings that determine expected returns."* (Daniel and Titman, 1997)

Score-based Portfolio Choice

- Applications in the financial industry assess factor exposure using rank scores
- Stocks are sorted with respect to characteristics associated with factors and these rank scores run into the portfolio-formation scheme
- In score-based portfolio choice stock characteristics are the essential primitive for portfolio selection

Contribution

- Scoring-based portfolio choice can be used to harvest significant return contributions
- The score-based portfolio choice can be represented analytically and there exists a tracking-error minimal portfolio path for increasing desired rank scores while holding other scores constant
- We analyse whether this return contribution offers a linear return premium
- Because scores are time-varying, introducing a heuristic for portfolio reallocation can further increase excess returns when transaction costs are large

Stock characteristics under consideration

- Characteristics are considered for the time period from October 1989 until November 2017 for the asset universe of the S&P 500
- Stock characteristics underlying the five factor model of Fama and French (2015)
- Weekly data lagged to account for Datastream backfill bias

Factor	Characteristic
Market Beta	Market sensitivity (MS)
Size	Market capitalization (MC)
Value	Book-to-market (BTM)
Profitability	Gross profits over total assets (GPOA)
Investment	Total Asset Growth (TAG)

Table: Factors and their respective characteristics

Method for return contribution

To measure the linear effect of the rank score on stock excess returns, we perform cross-sectional regressions of the excess returns on the stock characteristics:

$$r_{i,t} - r_f = \alpha_t + \sum_{c=1}^{m} \gamma_{c,t} s_{c,t,i} + \epsilon_t \qquad (1)$$

$r_{i,t}$	...	return of stock i from time t until $t+1$
r_f	...	return of risk-free asset
α_t	...	intercept of regression at time t
m	...	number of different asset characteristics / scores to be regarded in portfolio formation, $m \leq n$
$\gamma_{c,t}$	...	estimated return contribution of characteristic c at time t
$s_{c,t,i}$	...	score for characteristic c of stock i at time t
ϵ_t	...	error term at time t

Results Return Contribution

- We reject the null hypothesis that average γ over time is equal to zero: $H_0 : \bar{\gamma} = 0$ on the 90% confidence interval for BTM, GPOA and TAG
- In contrast, we accept our alternative $H_1 : \bar{\gamma} \neq 0$
- The signs of all return contributions support factor investing research, but we cannot reject H_0 for MS and MC
- The significance for GPOA and TAG is robust also for a longer time period of 4 weeks, whereas the p-value for BTM increases to 0.10-0.15 depending on the start week

	MS	MC	BTM	GPOA	TAG
$\bar{\gamma}$	0.0919	0.0635	0.1180	0.1373	0.0572
σ_γ	3.3920	1.6139	2.0349	1.6693	1.1673
p-Value	0.2998	0.1319	0.0266**	0.0017***	0.0608*

Table: Multivariate Regression Results Characteristics of FF5

Problem Statement

n	...	number of assets
S	...	$(n \times m)$ matrix of asset characteristics / scores, with full column rank, i.e., scores are linearly independent
Σ	...	$(n \times n)$ covariance matrix of asset returns, positive definite
$\mathbf{w}$	...	$(n \times 1)$ column vector of asset weights
$\mathbf{b}$	...	$(m \times 1)$ column vector of target characteristics / scores

- We search for the minimum-tracking error weights $\mathbf{w}$ (relative to some given weights $\mathbf{w}_0$) that satisfy some given portfolio characteristics / total score $\mathbf{b}$

Optimal Weights

The objective function of this problem is defined by:

$$\min_{\mathbf{w}} \left\{ \frac{1}{2}(\mathbf{w} - \mathbf{w}_0)'\Sigma(\mathbf{w} - \mathbf{w}_0) \right\},$$
$$\text{s.t.} \;\; S'\mathbf{w} = \mathbf{b}.$$

- Comment: Weights $\mathbf{w}_0$ and $\mathbf{w}$ do not necessarily sum to 1. Since the score matrix S has full column rank and Σ is positive definite, $\left(S'\Sigma^{-1}S\right)^{-1}$ exists.
- The optimal solution for $\mathbf{w}$ is found via Lagrange:

$$\text{(i)} \quad \mathbf{w}^* \; = \; \mathbf{w}_0 - \Sigma^{-1}S\left(S'\Sigma^{-1}S\right)^{-1}S'\mathbf{w}_0 + \Sigma^{-1}S\left(S'\Sigma^{-1}S\right)^{-1}\mathbf{b}$$

Score-mimicking Weights

Basis Weights B

The matrix B defined by

$$B \;=\; \Sigma^{-1} S \left(S' \Sigma^{-1} S \right)^{-1}, \quad (n \times m)$$

has full column rank and contains in its columns a basis of minimum-variance score-mimicking weight vectors. I.e., $B_{.,i}$ is the min-var weight vector with exposure 1 to the characteristic i and 0 to all others.

- Full column rank follows from initial assumptions on S and Σ.
- From (i) we observe that if $\mathbf{w}_0 = 0$, a target score $\mathbf{b}$ directly translates into weights

$$\mathbf{w}^* \;=\; B\mathbf{b}.$$

such that

$$S'\mathbf{w}^* \;=\; S'B\mathbf{b} \;=\; \mathbf{b}$$

Application of score-based portfolio choice

1. Start with equally-weighted benchmark portfolio
2. Construct matrix B and increase a desired score tracking-error minimal towards $b = 0.6$
3. Covariance matrix is constructed via

$$\Sigma = L\Sigma_f L' + \Omega$$

   where L represents the 52 weeks factor loading matrix, Σ_f the 52 weeks variance-covariance matrix for factor returns as downloaded from Kenneth French's website and Ω the covariance matrix of residuals (see e.g., Dangl, Randl, and Zechner, 2015)
4. This way also added factor sensitivities are controlled
5. Repeat this portfolio recomposition on a weekly basis

Portfolio specifics after tilt towards chosen characteristics

In a backtest study, we test the null hypothesis $H_0 : \Delta r_w = 0$ and reject this hypothesis for all characteristics but MS on the 95% confidence interval.

	$\bar{r}_w$	p-value Δr_w	$\sigma(r_w)$	$\bar{r}_a$	$\sigma(\bar{r}_a)$	$S(r_a, \sigma(\bar{r}_a))$
Equally-weighted	0.2448	NA	2.3741	10.4584	17.1195	0.6109
MS tilt	0.2604	0.1134	2.4845	11.3564	17.9157	0.6339
MC tilt	0.2501	0.0423**	2.4034	10.7605	17.3309	0.6209
BTM tilt	0.2578	0.0181**	2.3800	10.7605	17.1622	0.6270
GPOA tilt	0.2547	0.0018***	2.3688	11.0277	17.0816	0.6456
TAG tilt	0.2496	0.0215**	2.3681	10.7311	17.0769	0.6284

Table: Portfolio Statistics for tracking error minimization, with $b = 0.6$ for the respective score tilt

$\bar{r}_w$	...	average weekly return of tilted portfolio
Δr_w	...	difference in weekly portfolio returns to the benchmark
$\sigma(r_w)$	...	weekly standard deviation of portfolio returns
$\bar{r}_a$, $\sigma(r_a)$	...	annualized portfolio return and standard deviation
$S(\bar{r}_a, \sigma(r_a))$	...	Sharpe Ratio annualized

Linearity in the return contribution

1. For testing linearity, we take quintile subsets of the S&P 500 based on the investigated characteristics
2. Multivariate cross-sectional regressions are conducted analogously to before but with the restricted subsets
3. We analyse the return contribution from a given characteristic based on the magnitude of γ for its respective quintiles
4. We are particularly interested in the return contribution for characteristic c from its own quintile subsets

Return contribution for all characteristics from quintile subsets

We reject the null hypothesis $H_0 : \gamma_{Q1,BTM} = \gamma_{Q3,BTM}$ on the 90 % confidence interval:

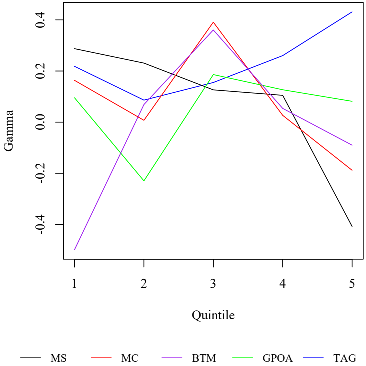

Figure: γ for respective quintile

How stable are these scores?

Transition Matrices show that also scores are time-varying:

- Transtion Matrix: For a starting decile given by its row, how many stocks (in percent) remain in this decile (column) after 52 weeks
- Example transition matrix for the market capitalization characteristic:

	1	2	3	4	5	6	7	8	9	10
1	0.8980	0.0949	0.0041	0.0013	0.0007	0.0003	0.0003	0.0002	0.0001	0.0002
2	0.1015	0.7084	0.1631	0.0179	0.0046	0.0021	0.0012	0.0008	0.0002	0.0001
3	0.0050	0.1646	0.5735	0.1977	0.0379	0.0124	0.0048	0.0021	0.0010	0.0009
4	0.0002	0.0200	0.2097	0.4636	0.2220	0.0568	0.0172	0.0067	0.0025	0.0014
5	0.0001	0.0054	0.0399	0.2225	0.4050	0.2331	0.0678	0.0178	0.0064	0.0019
6	0.0000	0.0026	0.0111	0.0622	0.2255	0.3937	0.2287	0.0553	0.0163	0.0046
7	0.0000	0.0007	0.0020	0.0130	0.0594	0.2022	0.3939	0.2527	0.0615	0.0147
8	0.0004	0.0003	0.0010	0.0037	0.0144	0.0502	0.1891	0.4246	0.2713	0.0450
9	0.0002	0.0000	0.0001	0.0010	0.0034	0.0086	0.0373	0.1855	0.5202	0.2437
10	0.0008	0.0002	0.0000	0.0001	0.0006	0.0020	0.0041	0.0207	0.1499	0.8217

How do time-varying scores affect portfolio allocation?

In a portfolio setting with transaction costs: When do we want to exchange stock A for stock B? We analyse in a backtest study.

- E.g., consider a portfolio that consists of the top decile stocks for a given characteristic c.
- We propose a trading hurdle that captures a desired portfolio return difference, such that reallocation is preferred:

$$\kappa < (1 + \Delta s_c \bar{\gamma}_c)^\tau - 1$$

- κ ... Trading Kosten
- τ ... Average Holding Period

How do we estimate the average holding period τ?

How do we estimate the average holding period τ?

After a 10-year burn-in period, at each portfolio formation period, we run a fixed-point iteration as follows:

1. Start with a top decile equally-weighted portfolio
2. Exchange every stock falling out of the portfolio
3. Compute the average holding period $\tilde{\tau}$ for this portfolio
4. Start with a new top decile equally-weighted portfolio
5. Reallocate only in cases when trading hurdle is reached
6. Restart with 3 until convergence is reached

$$\kappa < (1 + \Delta s_c \bar{\gamma}_c)^{\tau} - 1$$

We apply the fixed-point iteration to a portfolio setting

In a backtest simulation, we test our portfolio performance against a hard-to-beat benchmark: The equally-weighted top decile portfolio with immediate reallocation

- DeMiguel, Garlappi, and Uppal (2007) show that this portfolio is one of the hardest to beat consistently in terms of Sharpe ratio (i.e. risk-adjusted) and turnover
- DeMiguel, Garlappi, and Uppal (2007) use trading costs equivalent to 50 basis points per trade
- Edelen, Evans, and Kadlec (2013) show that per unit trading costs of mutual funds on average over all trading strategies sum up to 80 basis points
- Depending on the market capitalization of target stocks and strategies, trading costs range from 42 basis points to 164 basis points
- We run a backtest with 50, 100 and 150 basis points as per unit trading costs.

Backtest Results MS

Black: Log performance: Heuristic Rebalancing
Red: Log performance Benchmark portfolio (equally weighted decile portfolio)

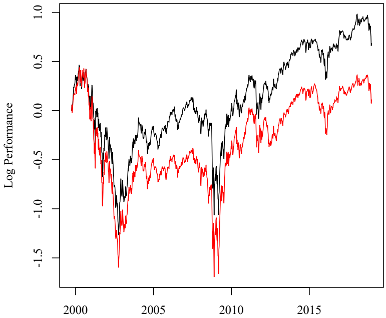

Figure: $\kappa = 50$ BP

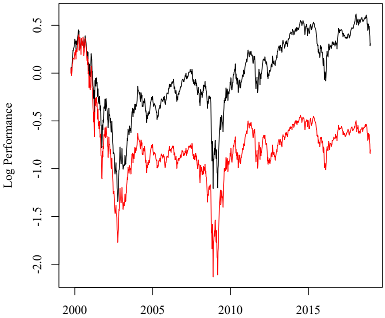

Figure: $\kappa = 150$ BP

Backtest Results MC

Black: Log performance: Heuristic Rebalancing
Red: Log performance Benchmark portfolio (equally weighted decile portfolio)

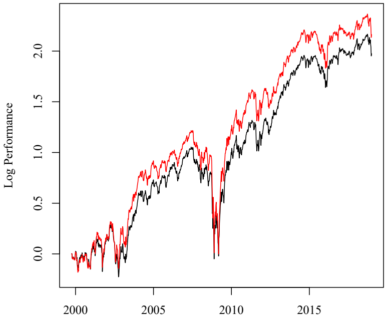

Figure: $\kappa = 50$ BP

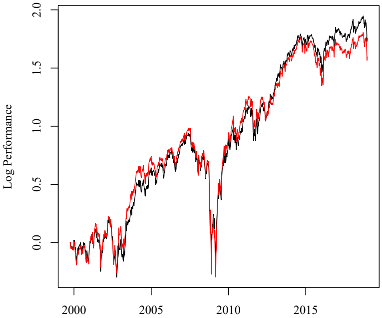

Figure: $\kappa = 150$ BP

Backtest Results BTM

Black: Log performance: Heuristic Rebalancing
Red: Log performance Benchmark portfolio (equally weighted decile portfolio)

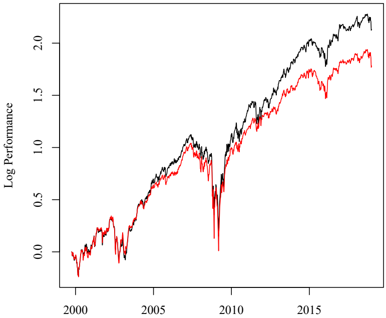

Figure: $\kappa = 50$ BP

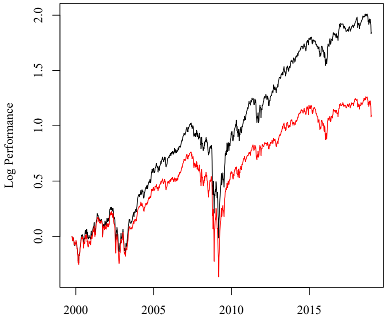

Figure: $\kappa = 150$ BP

Backtest Results GPOA

Black: Log performance: Heuristic Rebalancing
Red: Log performance Benchmark portfolio (equally weighted decile portfolio)

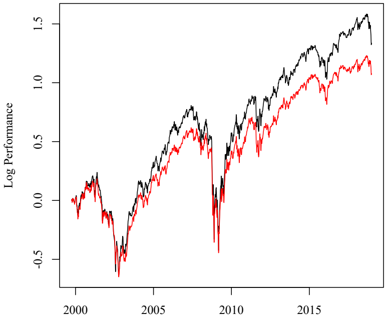

Figure: $\kappa = 50$ BP

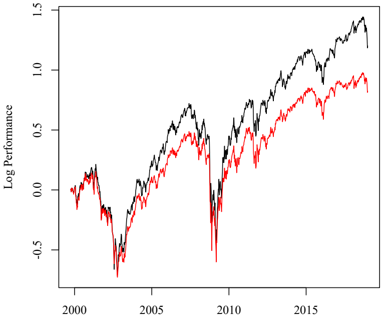

Figure: $\kappa = 150$ BP

Backtest Results TAG

Black: Log performance: Heuristic Rebalancing
Red: Log performance Benchmark portfolio (equally weighted decile portfolio)

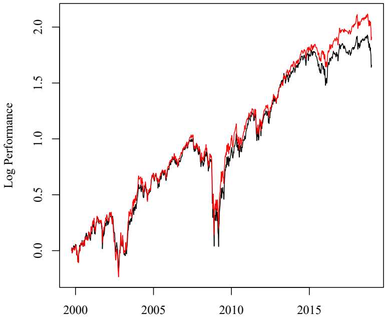

Figure: $\kappa = 50$ BP

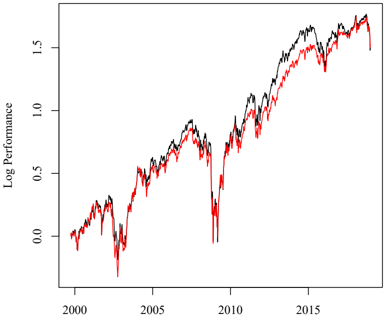

Figure: $\kappa = 150$ BP

Backtest Results Significance Tests

Return differences between the portfolio p following the trading hurdle heuristic and the benchmark portfolio b are denoted by $\Delta r^{\kappa}_{b,p}$

- We test the null hypothesis $H_0 : \Delta r^{\kappa}_{b,p} = 0$ that both portfolios achieve the same returns
- We cannot reject the null hypothesis for $\kappa = 50$ BP
- We do reject the null however for $\kappa = 150$ BP and find that returns are significantly different for MS and BTM on the 95% confidence interval, and for GPOA on the 90% confidence interval

	MS	MC	BTM	GPOA	TAG
t-statistic $\Delta r^{50}_{b,p}$	-0.9577	1.1946	-0.8339	-1.4829	0.8514
P-Value $\Delta r^{50}_{b,p}$	0.3385	0.2325	0.4045	0.1384	0.3948
t-statistic $\Delta r^{150}_{b,p}$	-2.3902	-0.1526	-1.9713	-1.9330	-0.0104
P-Value $\Delta r^{50}_{b,p}$	0.0170**	0.8787	0.0490**	0.0535*	0.9917

Conclusion

- We construct scores from stock characteristics and capture common factor risks in terms of score
- For these scores, we reject the null hypothesis that return contributions are equal to zero and find that scores contribute significantly to performance
- An increase in portfolio score, while holding other scores constant, brings about a significant increase in portfolio return for MC, BTM, GPOA and TAG
- We show an analytical solution for the tracking-error minimal increase of a desired portfolio score
- The linearity assumption holds with limitations particularly for the BTM score that need to be monitored in portfolio construction
- In a backtest study, we show that especially when transaction costs are large, regarding transaction costs can significantly increase portfolio returns.

References

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Linearity in the return contribution

1. Instead of an equally-weighted portfolio over the entire S&P 500, we now construct two equally weighted portfolios for the subset of Q1 and Q3
2. We increase the BTM score via matrix B until we touch the short-sale constraint, i.e. one of stocks has weight 0 and following path B would require a short sale
3. Re-run a backtest with weekly recomposition resulting in an average score increase of 0.02 for both quintile portfolios

Linearity in the return contribution

Black: Log performance increased BTM score
Red: Log performance Benchmark portfolio (equally weighted quintile portfolio)

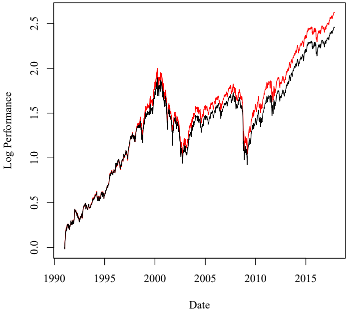

Figure: BTM tilt in BTM Q1

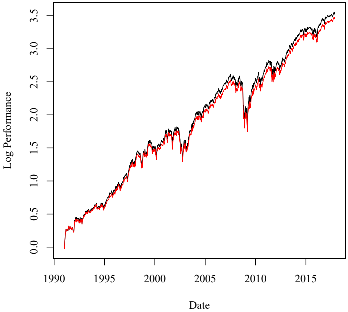

Figure: BTM tilt in BTM Q3

Robustness checks longer time period

	MS	MC	BTM	GPOA	TAG
$\bar{\gamma}_{M,1}$	0.2892	0.2832	0.2434	0.4944	0.3400
P-value $\bar{\gamma}_{M,1}$	0.3892	0.1365	0.1389	0.0045***	0.0499**
$\bar{\gamma}_{M,2}$	0.3093	0.2777	0.2367	0.4831	0.3311
P-value $\bar{\gamma}_{M,2}$	0.3702	0.1322	0.1592	0.0053***	0.0460**
$\bar{\gamma}_{M,3}$	0.3306	0.3000	0.2623	0.5281	0.3747
P-value $\bar{\gamma}_{M,3}$	0.3578	0.1399	0.1072	0.0016***	0.0234**
$\bar{\gamma}_{M,4}$	0.2214	0.2773	0.2676	0.5151	0.3682
P-value $\bar{\gamma}_{M,4}$	0.5042	0.1420	0.1011	0.0034***	0.0195**

Table: Multivariate Regression Results for 4 Week Rebalancing

Linearity

	Q1	Q2	Q3	Q4	Q5
$\bar{\gamma}_{MS,i}$	0.2874	0.2310	0.1262	0.1048	-0.4082
p-value $\bar{\gamma}_{MS,i} \neq 0$	0.1872	0.2438	0.5289	0.6621	0.4009
$\bar{\gamma}_{MC,i}$	0.1635	0.0072	0.3912	0.0266	-0.1883
p-value $\bar{\gamma}_{MC,i} \neq 0$	0.4155	0.9720	0.0634*	0.9030	0.5659
$\bar{\gamma}_{BTM,i}$	-0.4989	0.0687	0.3606	0.0538	-0.0901
p-value $\bar{\gamma}_{BTM,i} \neq 0$	0.0791*	0.7356	0.1140	0.8119	0.7642
$\bar{\gamma}_{GPOA,i}$	0.0953	-0.2297	0.1860	0.1267	0.0815
p-value $\bar{\gamma}_{GPOA,i} \neq 0$	0.7331	0.2950	0.3911	0.5636	0.7187
$\bar{\gamma}_{TAG,i}$	0.2183	0.0861	0.1550	0.2603	0.4314
p-value $\bar{\gamma}_{TAG,i} \neq 0$	0.4090	0.6887	0.4038	0.1681	0.0772*

Table: Significance tests for quintile portfolios